OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

COLLEGE OF ENGINEERING AND INFORMATICS

ENGINEERING MATHS QUALIFYING EXAMINATION 2019

First Paper

Time allowed: *Two* hours

Candidates for Computer Science & Information Technology and Project & Construction Management should take **4** questions out of 6. All other candidates should take **5** questions out of 6.

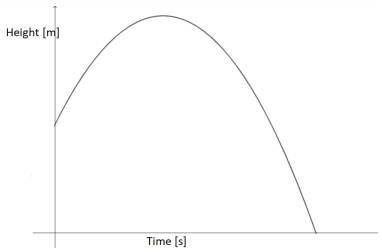
Formulae and Tables booklets are provided by the Exams Office
Calculators are permitted

1. **(a)** In an arithmetic sequence, the third term is $3\sqrt{2}+11$ and the sixth term is $3\sqrt{8}+20$. Find the first term and the common difference.

(a) An object is launched from the top of a building and its height above the ground from the moment it is launched can be modelled by the function

$$h(t) = -5t^2 + 20t + 20,$$

as shown in the diagram.



(i) Determine the height of the building.
(ii) Find the object's maximum height above the ground.
(iii) Calculate, to the nearest second, the time it takes for the object to reach the ground.

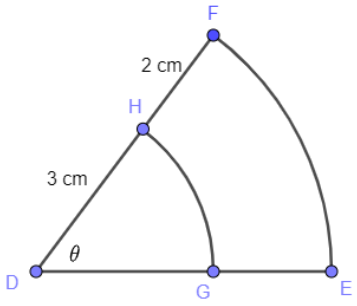
2. **(a)** The average speed s (in metres per second) at which a tsunami travels can be modelled by the equation

$$s = \sqrt{g \times d},$$

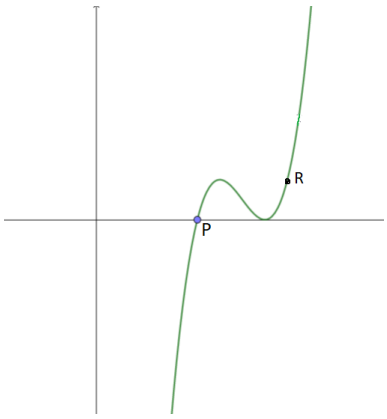
where $g = 9.8$ m/s^2 and d is the depth of the ocean (in metres) at the location of the earthquake.

A tsunami begins 400 km away from the coast where the depth of the ocean is 2000 m. Find (correct to the nearest minute) how long until this tsunami reaches the coast.

(b) Using the dimensions shown on the diagram below, find the value of θ, in radians, where the perimeter of the region $FHGE$ is 14 cm.



3. The equation of the curve given below is $f(x) = x^3 - 13x^2 + 55x - 75$, where $x \in \mathbb{R}$.



(a) Differentiate the function $y = f(x)$ and hence find the coordinates of the minimum and maximum turning points.

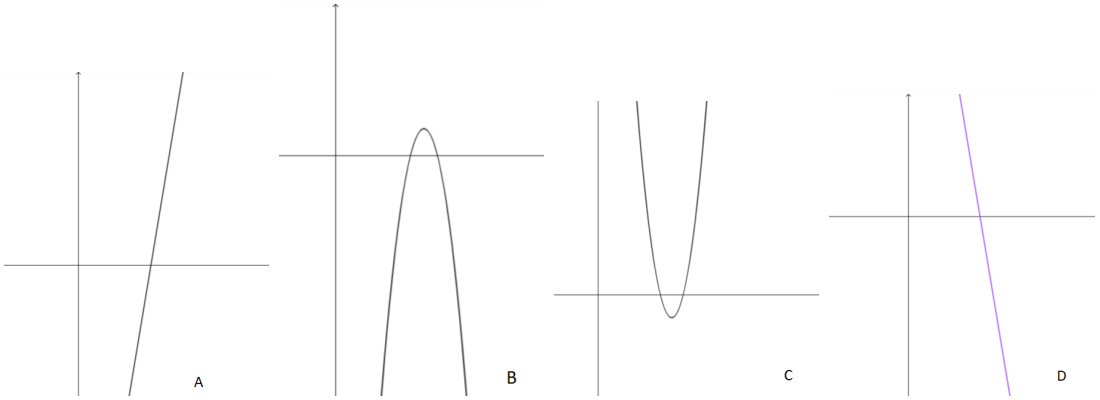
(b) Find the point of inflection.

(c) Show that the point $P(3, 0)$ belongs to the curve.

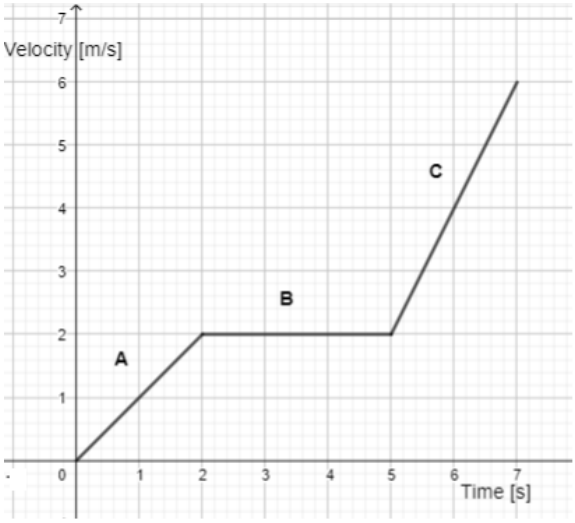
(d) Find the slope of the tangent to the curve at P.

(e) Find the coordinates of the point R on f at which the tangent is parallel to the tangent at P.

(f) From the graphs A, B, C, D below select the graph of the first derivative, $f'(x)$, and the graph of the second derivative, $f''(x)$.



4. **(a)** A velocity-time graph describing the motion of an object is shown below.



(i) The slope of a line on a velocity-time graph represents the acceleration of the object. What is the acceleration in each of the sections A, B and C above?

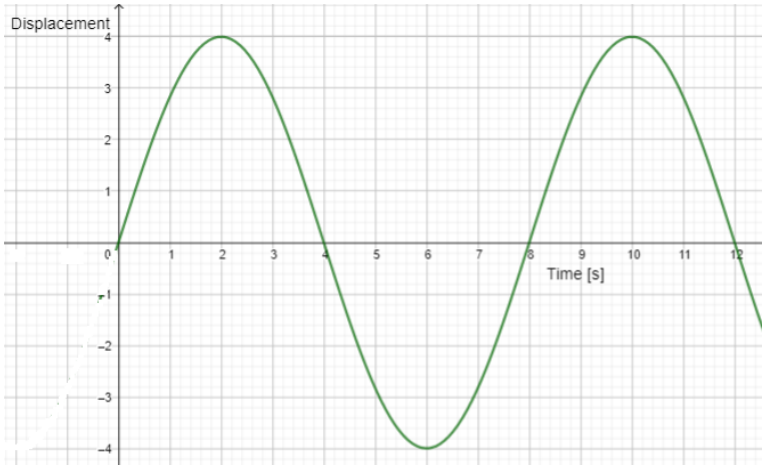
(ii) The area under this velocity-time graph represents the distance travelled. Find the total distance covered by the object from the graph above.

(b) Find the following integrals.

(i) $\displaystyle\int \left(e^{-x} + \frac{1}{x}\right) dx$

(ii) $\displaystyle\int_1^2 \left(x^2 + 3x - 5\right) dx$

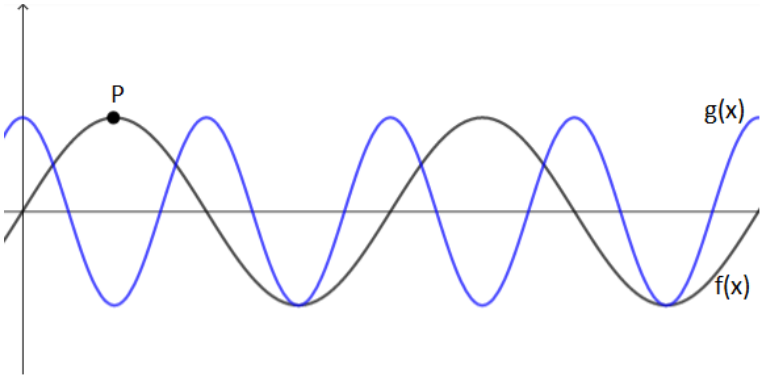
5. **(a)** The pendulum of a clock swings to the left and right of its resting position. Its periodic motion can be represented by the following trigonometric graph.



(i) What is the period of the pendulum's motion?

(ii) How many complete swings will the pendulum make in four minutes?

(b) The graphs of two trigonometric functions, $f(x) = a\sin(2x)$ and $g(x) = b\cos(kx)$, are shown in the diagram. The coordinates of P are $(45, 3)$, where the x coordinate is in degrees.



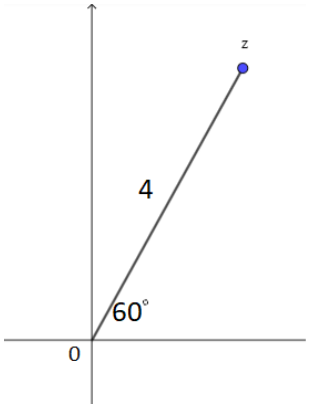
(i) Find the period and the range of $y = f(x)$ and write down the equation of $f(x)$.

(ii) Find the amplitude and the period of $y = g(x)$ and write down the equation of $g(x)$.

(iii) Use the graph to determine two values of x for which $f(x) = g(x)$.

6. **(a)** Write $\dfrac{2}{1-i}$ in the form $a + bi$, where $a, b \in \mathbb{R}$ and $i^2 = -1$.

(b) A complex number z is shown on the Argand diagram.



(i) Write the modulus and argument of z.

(ii) Write z in polar form.

(iii) Write z in the form $a + bi$, where $a, b \in \mathbb{R}$ and $i^2 = -1$.

(iv) Using de Moivre's theorem, or otherwise, evaluate z^{12}.

(c) Solve the equation $z^2 - 6z + 10 = 0$. Write your solutions in the form $a + bi$, where $a, b \in \mathbb{R}$ and $i^2 = -1$.