MP491: Nonlinear Systems (5 ECTS)

This course is an introductuion to the analysis of systems of nonlinear Ordinary Differential Equations (ODEs) and Maps.

Taught in Semester(s) II. **Examined** in Semester(s) II.

Workload: 36 hours (24 Lecture hours, 12 Tutorial hours).

Module Learning Outcomes. On successful completion of this module the learner should be able to:

1. Locate and calculate the stability for equilibria in 1-dim ODEs;
2. Locate and classify bifurcations for equilibria in 1-dim ODEs;
3. Locate, classify and calculate the stability for equilibria in linear 2-dim systems of ODEs;
4. Sketch phase-plane portraits about equilibria in linear 2-dim systems of ODEs;
5. Locate equilibria in nonlinear 2-dim systems of ODEs;
6. Linearise nonlinear 2-dim systems of ODEs, calculate the linear stability of equilibria and classify equilibria;
7. Sketch phase-plane portraits of nonlinear 2-dim systems of ODEs using iso-curves;
8. Analyse 2-dim Hamiltonian systems and sketch their phase-plane portraits;
9. Locate and classify Hopf bifurcations in nonlinear 2-dim systems of ODEs, and determine the stability of the corresponding limit cycles;
10. Locate and calculate the stability for fixed points and periodic orbits in 1-dim nonlinear maps;
11. Locate bifurcations in 1-dim nonlinear maps;
12. Describe period-doubling cascades to chaos in 1-dim nonlinear maps.
13. Ability to use Python Jupyter notebooks to numerically draw streamlines.

Indicative Content

This course is concerned with systems of nonlinear Ordinary Differential Equations (ODEs) and Maps. Topics covered include:

1. 1-dimensional differential equations: equilibria, stability, bifurcations;
2. 2-dimensional linear systems of ODEs: equilibria, stability, phase-plane portraits;
3. 2-dimensional nonlinear systems of ODEs: equilibria, linearisation, linear stability, phase-plane portraits;
4. 2-dimensional Hamiltonian systems: equilibria, stability, phase-plane portraits;
5. Limit cycles: Hopf bifurcations, stability;
6. 1-dimensional difference equations and maps cycles: fixed points, periodic orbits, stability, bifurcations.

Module Resources

Strogatz Steven H., Nonlinear Dynamics and Chaos: with Applications to Physics, Biology, Chemistry, and Engineering, Westview Press, 1994,

Kuznetsov Yu.A, Elements of Applied Bifurcation Theory – Third Edition, Springer, 2010.
